

Effective theories of confinement*

L. Dittmann^a[TPI]Theoretisch–Physikalisches Institut, Friedrich–Schiller–Universität Jena,
Max–Wien–Platz 1, 07743 Jena, Germany, T. Heinzl^{bc†‡}, A. Wipf^d[TPI]

^a[

^cSektion Physik, Ludwig–Maximilians–Universität, Theresienstraße 37, 80333 München, Germany

We review some approaches to describe confinement in terms of effective (model) field theories. After a brief discussion of the dual Abelian Higgs model, we concentrate on a lattice analysis of the Faddeev–Niemi effective action conjectured to describe the low-lying excitations of $SU(2)$ gluodynamics. We generalize the effective action such that it contains all operators built from a unit color vector field n with $O(3)$ symmetry and maximally four derivatives. To avoid the presence of Goldstone bosons, we include explicit symmetry breaking terms parametrized by an external field h of mass-dimension two. We find a mass gap of the order of 1.5 GeV.

1. INTRODUCTION

Let us begin by recalling the definition of confinement: in *pure* Yang–Mills theory, the potential between static quarks in the fundamental representation ($t = 1/2$ for $SU(2)$) grows *linearly* at large interquark distances, $R \gtrsim 1$ fm,

$$V_{Q\bar{Q}} = \sigma_{1/2} R. \quad (1)$$

The constant of proportionality is the (fundamental) string tension, $\sigma_{1/2} \simeq (450 \text{ MeV})^2$. The definition above is equivalent to the statement that the Wilson loop decays with an area law.

For static sources in higher representations ($t = 1, 3/2, \dots$) one has *screening* for large R . For intermediate distances, however, there is again a linear rise, $V_t = \sigma_t R$, where the higher string tensions obey ‘Casimir scaling’ [1–3],

$$\sigma_t = C_t \sigma \equiv t(t+1)\sigma, \quad (2)$$

A potential V_t proportional to the quadratic Casimir C_t is, of course, well known from perturbation theory. The one-gluon exchange potential is $V_{\text{OGE}} \sim (\text{charge})^2 \sim T^a T^a$. It is, however, unclear why such a behavior should also be true at larger distances, i.e. in the nonperturbative regime.

The simplest model explanation of confinement is the dual-superconductor scenario [4,5]. This views the QCD vacuum as a monopole condensate, for which a dual Meissner effect leads to a (chromo)electric flux tube between static quarks.

Apart from (static) quark confinement there should also be gluon confinement implying a finite range of the gluonic interactions, i.e. a mass gap. The connection between the linear potential and the existence of a mass gap is somewhat elusive. A qualitative Peierls type argument goes as follows [6]. The partition function for magnetic flux lines of length L has an energy–entropy behavior according to

$$Z \sim \exp(-cL/e^2 + L \ln c'), \quad (3)$$

where we have used electric–magnetic duality in writing the magnetic coupling as $1/e$. Energy–entropy balance suggests a phase transition (monopole condensation) if $e \sim 1$ leading to ‘topological order’ [7]. This implies *disorder* of the dual objects which, roughly speaking, should be the ‘glue’. Thus, there is no long-range order in the gluonic sector so that there must be a mass gap.

2. EXAMPLES

In this section we will discuss two alternative approaches, emphasizing either linear confinement or the presence of a mass gap.

*Talk presented by T.H.

[†]heinzl@theorie.physik.uni-muenchen.de

[‡]supported in part by DFG

2.1. Dual Abelian Higgs model (DAHM)

This model [8,9] is basically a field theoretical Ginzburg–Landau realization of the dual superconductor. Thus, the Lagrangian is

$$\mathcal{L} = -\frac{1}{4g^2}F^2 + |D\phi|^2 + \lambda(|\phi|^2 - \phi_0^2)^2. \quad (4)$$

the ingredients being a dual photon coupled to a magnetically charged Higgs field, ϕ . Dual superconductivity is achieved by the familiar Higgs mechanism. The Higgs vacuum expectation value, $\langle\phi\rangle \equiv \phi_0$, provides the scale for both the photon and Higgs mass, $m_\gamma^2 = 2g^2\phi_0^2$, $m_H^2 = 4\lambda\phi_0^2$. Note that with ϕ_0^2 there is a new operator of mass-dimension two in the game [10].

The model has a classical soliton solution, the Abrikosov–Nielsen–Olesen (ANO) vortex, which has finite energy per unit length implying a linear potential, $V(L) = \sigma L$. The string tension σ is proportional to ϕ_0^2 .

Interestingly, it is possible to derive a string representation of the ANO vortex [11–13], for a review see [14]. One obtains a Nambu–Goto action with higher curvature terms added.

Like any model, the DAHM also has its problems. Being Abelian, it actually describes a $U(1)$ confinement. This can be viewed as stemming from an Abelian projection of Yang–Mills theory [15], where the $SU(2)$ gauge freedom has been partially fixed down to a $U(1)$ subgroup. For this reason there is no confinement of objects that are uncharged (neutral) with respect to this $U(1)$, for example the diagonal gluons. In addition, it turns out that there is no Casimir scaling in this model. This has been rectified only recently [16]. Finally, it seems difficult to describe the gluonic sector in the DAHM. Attempts to represent glueballs as closed vortices are still in their infancy [17]. Thus, it is unclear how to obtain the mass gap in the glueball spectrum. The second model we are going to discuss is meant to do better in this particular respect.

2.2. Faddeev–Niemi action

Recently, Faddeev and Niemi (FN) have suggested that the infrared sector of Yang–Mills theory might be described by the following low-

energy effective action [18],

$$S_{\text{FN}} = \int d^4x \left[m^2 (\partial_\mu \mathbf{n})^2 + \frac{1}{4e^2} (\mathbf{n} \cdot \partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n})^2 \right]. \quad (5)$$

Here, \mathbf{n} is a unit vector field with values on S^2 , $\mathbf{n}^2 \equiv n^a n^a = 1$, $a = 1, 2, 3$; m^2 is a dimensionful and e a dimensionless coupling constant. The FN ‘field strength’ is defined as

$$H_{\mu\nu} \equiv \mathbf{n} \cdot \partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n}. \quad (6)$$

FN claim that (5) “is the *unique* local and Lorentz-invariant action for the unit vector \mathbf{n} which is at most quadratic in time derivatives so that it admits a Hamiltonian interpretation and involves *all* such terms that are either relevant or marginal in the infrared limit”.

It has been shown that S_{FN} supports string-like knot solitons [19,20], characterized by a topological charge which equals the Hopf index of the map $\mathbf{n} : S^3 \rightarrow S^2$. In analogy with the Skyrme model, the H^2 term is needed for stabilization. The knot solitons can possibly be identified with gluonic flux tubes and are thus conjectured to correspond to glueballs. For a rewriting in terms of curvature free $SU(2)$ gauge fields and the corresponding reinterpretation of S_{FN} we refer to [21].

In order for the model to really make sense, however, the following problems have to be solved. First of all, neither the interpretation of \mathbf{n} nor its relation to Yang–Mills theory have been clarified. An analytic derivation of the FN action requires an appropriate change of variables, $A \rightarrow (\mathbf{n}, X)$, which decomposes the Yang–Mills potential A into (a function of) \mathbf{n} and some remainder X . Although progress in this direction has been made [22–25], there are no conclusive results up to now.

Second, there is no reason why both operators in the FN ‘Skyrme term’, which can be rewritten as

$$H^2 = (\partial_\mu \mathbf{n})^4 + (\partial_\mu \mathbf{n} \cdot \partial_\nu \mathbf{n})^2, \quad (7)$$

should have the same coupling. Third, and conceptually most important, S_{FN} has the same spontaneous symmetry breaking pattern as the non-linear σ -model, $SU(2) \rightarrow U(1)$. Hence, it should admit two Goldstone bosons and one expects to find *no* mass gap.

We have scrutinized the FN action using lattice methods. To this end we made a sufficiently general ansatz for an \mathbf{n} -field action that contains (5) as a special case. In particular, we allow for explicit symmetry breaking terms to avoid the appearance of Goldstone bosons.

3. LATTICE TEST

After generating $SU(2)$ lattice configurations using the standard Wilson action we fix to a covariant gauge following the continuum approach of [24,25]. We chose Landau gauge (LG), defined by maximizing $\sum_{x,\mu} \text{tr} \Omega U_{x,\mu}$ w.r.t. the gauge transformation Ω , leaving a residual global $SU(2)$ -symmetry. The field \mathbf{n} is then obtained via maximizing the functional $F_{\text{MAG}} \equiv \sum_{x,\mu} \text{tr} (\tau_3 {}^g U_{x,\mu} \tau_3 {}^g U_{x,\mu}^\dagger)$ of the maximally Abelian gauge (MAG) [15,26]. This yields a gauge transformation g which we use to define our \mathbf{n} -field,

$$n_x = g_x^\dagger \tau_3 g_x . \quad (8)$$

It is important to note that this definition leaves a residual local $U(1)$ unfixed.

Since the configurations generated originally are randomly distributed along their orbits, the gauge fixing is absolutely crucial for rendering the definition (8) gauge invariant [27]. This is illustrated in Fig. 1.

Our ansatz for the effective action is $S_{\text{eff}} = \sum_i \lambda_i S_i[\mathbf{n}]$ with couplings λ_i and operators S_i . Up to fourth order in a gradient expansion there are the symmetric terms

$$(\nabla_\mu \mathbf{n})^2, (\square \mathbf{n})^2, (\nabla_\mu \mathbf{n})^4, (\nabla_\mu \mathbf{n} \cdot \nabla_\nu \mathbf{n})^2, \quad (9)$$

and the symmetry breaking terms including a ‘source field’ \mathbf{h} ,

$$\mathbf{n} \cdot \mathbf{h}, (\mathbf{n} \cdot \mathbf{h})^2, (\nabla_\mu \mathbf{n})^2 \mathbf{n} \cdot \mathbf{h}. \quad (10)$$

The couplings λ_i can be obtained by use of an inverse Monte Carlo method [28]: rotational invariance of the functional measure implies an infinite set of Schwinger–Dyson equations providing an overdetermined linear system for the couplings,

$$\sum_j \langle F_i^{ab}[\mathbf{n}] S_{,b}^j[\mathbf{n}, \mathbf{h}] \rangle \lambda_j = \langle I_i^a[\mathbf{n}] \rangle. \quad (11)$$

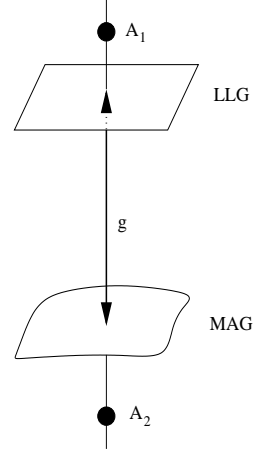


Figure 1. Gauge invariant definition of $n \equiv g^\dagger \tau_3 g$. The gauge equivalent configurations A_1 and A_2 are both mapped onto the same ‘representatives’ on the lattice LG or MAG slices (ignoring Gribov copies). Thus, they are both associated with the *same* gauge transformation g defining n .

Here, F_i^{ab} and I_i^a are known functions of \mathbf{n} , typically linear combinations of \mathbf{n} -point functions.

All computations have been done on a 16^4 -lattice with Wilson coupling $\beta = 2.35$, lattice spacing 0.13 fm and periodic boundary conditions. For the LG we used Fourier accelerated steepest descent [29]. The MAG was achieved using two independent algorithms, one (AI) being based on ‘geometrical’ iteration [30], the other (AII) analogous to LG fixing (see Fig. 2).

As expected, we observe a non-vanishing expectation value of the field in the three-direction that can be thought of as a ‘magnetization’ \mathfrak{M} , $\langle n^a \rangle = \mathfrak{M} \delta^{a3}$. Thus, the global symmetry is broken explicitly according to the pattern $SU(2) \rightarrow U(1)$. This also shows up in the behavior of the two-point functions (Fig. 3), which exhibit clustering, $\langle n^3(0) n^3(x) \rangle \sim \langle n^3 \rangle \langle n^3 \rangle = \mathfrak{M}^2$, for large distances. Furthermore, the transverse correlation function (of the would-be Goldstone bosons)

$$G^\perp(x) \equiv \frac{1}{2} \langle n^i(0) n^i(x) \rangle, \quad i = 1, 2, \quad (12)$$

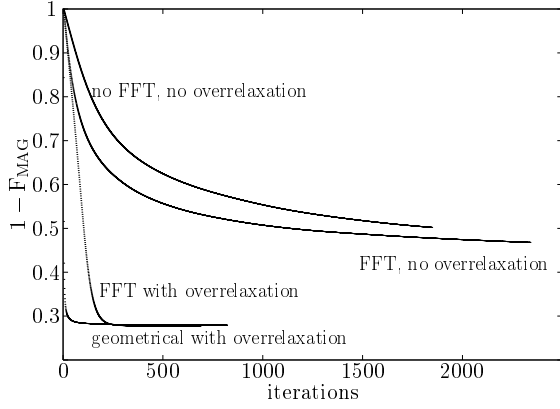


Figure 2. Behavior of the MAG-functional using different algorithms.

decays exponentially as shown in Fig. 4. This means that there is a nonvanishing mass gap M whose value can be obtained by a fit to a cosh-function.

The numerical values of the observables, \mathfrak{M} , M and the transverse susceptibility, $\chi^\perp \equiv \sum_x G^\perp(x)$, are summarized in Table 1 for both algorithms: The slight disagreement between AI

Table 1
Numerical values for some observables (all numbers in units of the lattice spacing).

	\mathfrak{M}	χ^\perp	M
AI	0.436	0.636	0.95
AII	0.352	0.596	1.01

and AII is expected from our still somewhat low statistics. The numerical results for the mass gap M lead to a value of about 1.5 GeV in physical units.

A first check of our method is to consider the minimal ansatz consisting of the first (leading) terms of (9) and (10), respectively,

$$S_{\text{eff}} = \sum_x (\lambda_1 (\nabla_\mu \mathbf{n}_x)^2 + \lambda_2 \mathbf{n}_x \cdot \mathbf{h}/|\mathbf{h}|) , \quad (13)$$

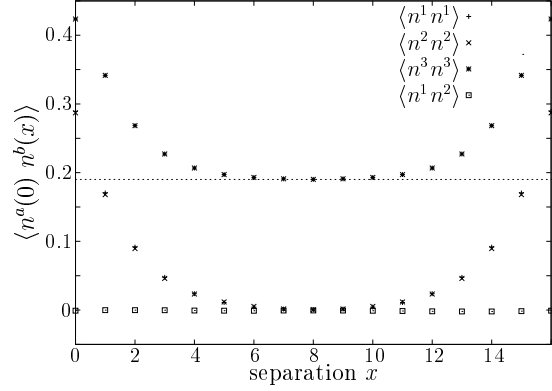


Figure 3. Two-point correlators of the field \mathbf{n} obtained via algorithm AI. The dotted line represents the (squared) VEV of \mathbf{n} , $\langle n^3 \rangle^2 = \mathfrak{M}^2$. The same behavior is obtained via AII with slightly different plateau value (see Table 1).

where $\lambda_2 \equiv \lambda_1 |\mathbf{h}|$. We thus have two couplings, λ_1 and $|\mathbf{h}|$, the latter representing an alternative new operator of mass-dimension two. Within the ansatz (13), it can be determined using an exact lattice Ward identity and the data from Table 1, $|\mathbf{h}| = \mathfrak{M}/\chi^\perp \simeq (1.3 \text{ GeV})^2$. (14)

In terms of the mass gap, on the other hand, one has

$$|\mathbf{h}| = M^2 + O(\lambda_1^{-1}) = (1.5 \text{ GeV})^2 + O(\lambda_1^{-1}), \quad (15)$$

so that we find qualitative agreement already to leading order in the derivative expansion.

The effective couplings have to be determined by solving (11). Results already obtained will be reported elsewhere.

4. CONCLUSIONS

We have discussed two effective theories meant to describe the confinement dynamics of pure Yang-Mills theory, the dual Abelian Higgs and the Faddeev-Niemi model. The latter has been modified by allowing for symmetry breaking terms in order to avoid the appearance of Goldstone modes. Both models then contain new operators of mass-dimension two, the vacuum expectation value of the Higgs field (squared), or

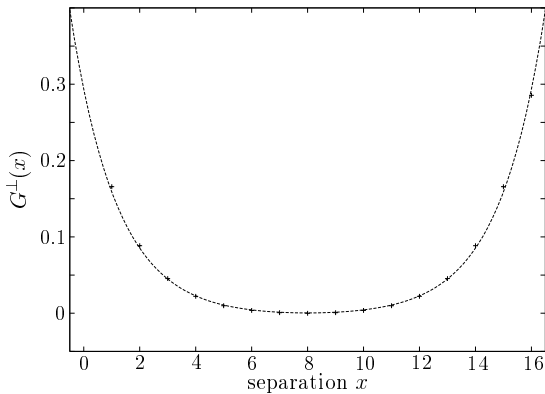


Figure 4. The transverse correlation function G^\perp , fitted to $G^\perp(x) \sim \cosh(-M(x - L/2))$.

an external ‘source’ field h , respectively. The former implies a non-vanishing string tension, the latter a mass gap.

At the moment, the relation between the two alternative descriptions is unclear. One may speculate that some kind of duality could unify them.

Acknowledgements

The authors are indebted to S. Shabanov for suggesting this investigation and to P. van Baal for raising the issue of Goldstone bosons. T.H. thanks A. Bassetto and his team for organizing such a stimulating meeting. Discussions with P. van Baal, P. de Forcrand, E. Seiler and M. Teper are gratefully acknowledged.

REFERENCES

1. R. P. Feynman, in: Weak And Electromagnetic Interactions At High Energies, Les Houches Proceedings 1976, North-Holland, Amsterdam 1977.
2. J. Ambjorn, P. Olesen and C. Peterson, Nucl. Phys. B240 (1984) 189, 533.
3. G.S. Bali, Nucl. Phys. Proc. Suppl. 83 (2000) 422.
4. G. ‘t Hooft, in: High Energy Physics, Proceedings of the EPS International Conference, Palermo 1975, A. Zichichi, ed., Editrice Compositori, Bologna 1976.
5. S. Mandelstam, Phys. Rept. 23 (1976) 245.
6. A.M. Polyakov, Gauge Fields and Strings, Harwood Academic, Chur 1987.
7. J.M. Kosterlitz and D.J. Thouless, J. Phys. C6 (1973) 1181.
8. M. Baker, J.S. Ball, F. Zachariasen, Phys. Lett. B152 (1985) 351.
9. S. Maedan and T. Suzuki, Progr. Theor. Phys. 81 (1989) 229.
10. E.V. Shuryak, hep-ph/9911244.
11. D. Förster, Nucl. Phys. B81 (1974) 84.
12. J.L. Gervais and B. Sakita, Nucl. Phys. B91 (1975) 301.
13. P. Orland, Nucl. Phys. B428 (1994) 221.
14. D. Antonov, Surveys High Energ. Phys. 14 (2000) 265.
15. G. ‘t Hooft, Nucl. Phys. B190 (1981) 455.
16. M.N. Chernodub, F.V. Gubarev, M.I. Polikarpov and V.I. Zakharov, hep-lat/0103033.
17. Y. Koma, H. Suganuma and H. Toki, Phys. Rev. D60 (1999) 074024.
18. L. Faddeev and A.J. Niemi, Phys. Rev. Lett. 82 (1999) 1624.
19. L. Faddeev and A.J. Niemi, Nature 387 (1997) 58.
20. R.A. Battye and P.M. Sutcliffe, Phys. Rev. Lett. 81 (1998) 4798.
21. P. van Baal and A. Wipf, Phys. Lett. B 515 (2001) 181.
22. E. Langmann and A.J. Niemi, Phys. Lett. B463 (1999) 252.
23. S.V. Shabanov, Phys. Lett. B458 (1999) 322.
24. S.V. Shabanov, Phys. Lett. B463 (1999) 263.
25. H. Gies, Phys. Rev. D63 (2001) 125023.
26. A.S. Kronfeld et al., Phys. Lett. B198 (1987) 516.
27. P. de Forcrand, private communication.
28. M. Falcioni et al., Nucl. Phys. B265 (1986) 187.
29. C.T.H. Davis et al., Phys. Rev. D37 (1988) 1581.
30. G. Bali, private communication.